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Accelerating black hole chemistry

Ruth Gregory [a,b,*], Andrew Scoins [a]

[a] Centre for Particle Theory, Department of Mathematical Sciences and Department of Physics, Durham University, South Road, Durham, DH1 3LE, UK
[b] Perimeter Institute, 31 Caroline St., Waterloo, Ontario, N2L 2Y5, Canada

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ABSTRACT

We introduce a new set of chemical variables for the accelerating black hole. We show how these expressions suggest that conical defects emerging from a black hole can be considered as true hair – a new charge that the black hole can carry – and discuss the impact of conical deficits on black hole thermodynamics from this 'chemical' perspective. We conclude by proving a new *Reverse Isoperimetric Inequality* for black holes with conical defects.

Black hole thermodynamics has seen a rich renaissance over the past decade, as the understanding of the role of thermodynamic pressure has been fleshed out and explored [1–8]. A key conceptual development was understanding that the mass term in the First Law of thermodynamics, related to the mass parameter m in the Newtonian potential of the black hole, was not in fact the internal energy of the black hole, but rather its enthalpy [2], i.e. the natural first law for a black hole with a cosmological constant includes not only the charges of electromagnetism and rotation, but also the impact of the non-zero energy coming from the cosmological constant in the volume inside the black hole:

$$dM = TdS + VdP + \Omega dJ + \Phi dQ\,. \tag{1}$$

Once one includes the possibility of a varying pressure, black hole thermodynamics more naturally resembles conventional thermodynamics, not only in its differential sense, but also in its integrable sense: the ideal gas relations $dU = TdS - PdV$, $U = c_V PV$ have their counterpart in the differential first law (1) and an integral Christodoulou-Ruffini [9,10] relation, that for four-dimensional Kerr-Newman-AdS black holes reads

$$M^2 = \frac{S}{4\pi}\left[1+\frac{\pi Q^2}{S}+\frac{8PS}{3}\right]^2 + \frac{4\pi^2 J^2}{S}\left[1+\frac{8PS}{3}\right], \tag{2}$$

and can be massaged into an ideal-gas like relation at large volume/entropy. Although the formulae for the enthalpy, charges and potentials are naturally derived from the black hole geometry, and written in terms of the metric parameters and horizon radius, expressing the thermodynamic potentials and enthalpy purely in terms of extensive quantities (such as explored in [5,7]) allows a natural identification with classic thermodynamics, and elucidates the *chemical* nature of the phase space of black holes.

There is another reason to specify the closed form of the black hole charges and enthalpy: While it is possible for material systems to have many charges and chemical potentials, the black hole on the other hand is believed to carry only mass, charge and angular momentum, a situation summed up by the *no-hair theorems* [11,12], and while these are now understood in a broader context to be somewhat limited, the basic picture from the perspective of classic black hole thermodynamics was that thermodynamic charges were still narrowly restricted, $M = M(S, P, J, Q)$. Recently however, a new type of 'charge' for a black hole has been explored and added to this stable: a conical deficit, μ [13–17], often interpreted as a cosmic string, that can either run symmetrically along the axis of the black hole [18–21], or have different values along the North and South axes, leading to an *accelerating* black hole, encoded by the C-metric [22,23] (and including a negative cosmological constant $\Lambda = -3/\ell^2$):

$$\begin{aligned} ds^2 = \frac{f(r)}{\Sigma H^2}\left[\frac{dt}{\alpha} - a\sin^2\theta\frac{d\varphi}{K}\right]^2 - \frac{\Sigma dr^2}{f(r)H^2} - \frac{\Sigma r^2}{g(\theta)H^2}d\theta^2 \\ - \frac{g(\theta)\sin^2\theta}{\Sigma r^2 H^2}\left[\frac{adt}{\alpha} - (r^2+a^2)\frac{d\varphi}{K}\right]^2 \end{aligned} \tag{3}$$

where the usual Schwarzschild potential $1 - 2m/r$ is augmented not only by the charge and angular momentum terms, but also by an "acceleration" parameter, A:

* Corresponding author.
E-mail addresses: r.a.w.gregory@durham.ac.uk (R. Gregory), andrew.d.scoins@durham.ac.uk (A. Scoins).

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$$
\begin{aligned}
f(r) &= (1-A^2r^2)\left[1-\frac{2m}{r}+\frac{a^2+e^2}{r^2}\right]+\frac{r^2+a^2}{\ell^2}\,,\\
g(\theta) &= 1+2mA\cos\theta+(\Xi-1)\cos^2\theta\,,\\
\Sigma &= 1+\frac{a^2}{r^2}\cos^2\theta\,, \qquad H=1+Ar\cos\theta\,,\\
\Xi &= 1+e^2A^2-\frac{a^2}{\ell^2}(1-A^2\ell^2)\,.
\end{aligned}
\tag{4}
$$

Note that the black hole is assumed to spin on its axis, the acceleration term modifying the angular parts of the metric, distorting the sphere to a teardrop, with a conical deficit (at least) at one of the poles. This deficit is revealed by taking the limit of the metric as we approach each pole, and is encoded by the *tension*

$$
\mu_\pm = \frac{1}{4}\left[1-\frac{\Xi\pm 2mA}{K}\right]. \tag{5}
$$

(with '+' corresponding to the North Pole, and '−' the South) interpreted as a cosmic string emerging from the black hole, causing it to accelerate.

Usually, an object under uniform acceleration will have an acceleration horizon, as ultimately an accelerating object will asymptote the speed of light, however, in AdS spacetime, the negative curvature of the space means that an object at fixed finite displacement from the centre of the spacetime is actually undergoing uniform acceleration. This means that a black hole suspended from the boundary by a cosmic string is indeed accelerating, even though the sole horizon is that of the black hole. This régime of accelerating black hole solutions that are truly static with respect to an observer at the boundary are called *slowly accelerating* black holes [24]. Roughly speaking, the criterion for slow acceleration is that the scale set by acceleration, A^{-1} is much larger than the AdS radius, $A\ell < 1$, although the true limit is dependent on the mass, charge and angular momentum of the black hole. As the acceleration increases, the position of the suspended black hole moves closer to the boundary until at $A\ell \sim 1$ there is a shift in the global structure of the spacetime and for $A\ell > 1$, the black hole now accelerates in from, and out to, the AdS boundary, see [25] for a discussion of the causal structure of the C-metric.

In a series of papers, [13–17] (see also [26,27]) the thermodynamics of conical deficits and accelerating black holes was explored and refined. The key insight was to use tools from holographic renormalization to properly calculate the various charges of the slowly accelerating black hole spacetime [16,17]. The nett result is a set of thermodynamic variables for the black hole, expressed in terms of the black hole metric parameters and the horizon radius r_+, that include the conical deficit as a charge, and introduce the conjugate chemical potential, a *thermodynamic length*, λ. The First Law has full cohomogeneity:

$$
dM = TdS+\Phi dQ+\Omega dJ+\lambda_+ d\mu_+ +\lambda_- d\mu_- + VdP\,, \tag{6}
$$

with all the physical parameters of the geometry corresponding to a thermodynamic charge. One of the key difficulties in determining the correct enthalpy was in properly identifying the time-like Killing vector for determining the mass of the black hole. The slowly accelerating black hole, being at a fixed point from the boundary, has the same time coordinate (up to a factor) as the asymptotic AdS spacetime, and the mass can be found via a holographic renormalization procedure. The resulting enthalpy thus contains factors dependent on the acceleration parameter:

$$
M = \frac{m}{K\Xi}\sqrt{(\Xi+a^2/\ell^2)(1-A^2\ell^2\Xi)} \tag{7}
$$

that then propagate throughout the expressions for thermodynamic volume and length. The accelerating black hole also obeys a Smarr relation [28]

$$
M = 2(TS+\Omega J-PV)+\Phi Q\,, \tag{8}
$$

that does not contain any trace of acceleration or tension.

There seems to be puzzle therefore: The first law has full cohomogeneity, yet the Smarr relation has no tension. Further, while the expressions obtained in [17] are perfectly adequate for the implicit study of black hole thermodynamics, the 'chemical' nature of the black hole is less transparent, and typically has to be studied numerically, and parametrically, in terms of the horizon radius r_+ that, for example, tracks entropy. To elucidate the *chemical* nature of the black hole, and to allow a more general analytic analysis of the phase space our aim is therefore to have closed-form expressions, such as (2), i.e. an integral expression of the form $M^2(S,P,Q,J,\mu_\pm)$, together with expressions for the chemical potentials in the form $\phi_i = \partial M/\partial q_i$, where q_i stands for a charge, $S,P,Q,J,\mu_\pm$ and ϕ_i its corresponding potential $T,V,\Phi,\Omega,\lambda_\pm$. Given that the Smarr relation is a statement about scaling dimension, and is given in terms of charges and potentials, it does not preclude an expression for M that includes the tensions.

The new physics in the accelerating black hole is that of the conical deficit(s), and while the individual tensions are natural geometric variables, they do not distinguish between an overall conical deficit, such as the cosmic string threading a black hole (that does not have issues with a slow acceleration limit) and a differential conical deficit that produces a nett force on the black hole, inducing acceleration. From the perspective of black hole chemistry, it turns out that the conical deficits are more conveniently encoded in the average and differential conical deficits of the spacetime:

$$
\begin{aligned}
\Delta &= 1-2(\mu_+ +\mu_-) = \frac{\Xi}{K}\,,\\
C &= \frac{(\mu_- -\mu_+)}{\Delta} = \frac{mA}{K\Delta} = \frac{mA}{\Xi}\,.
\end{aligned}
\tag{9}
$$

The tensions are bounded below by requiring positivity of energy (or tension) and above by the fact that the maximal conical deficit is 2π. With $A \ge 0$, so that $0 \le \mu_+ \le \mu_- \le 1/4$, this translates into $0 \le C \le \mathrm{Min}\left\{\frac{1}{2}, \frac{1-\Delta}{2\Delta}\right\}$. Although Δ and C are not unconstrained – introducing an acceleration necessarily also introduces an overall average deficit – it proves to be the best way to express the impact of the conical deficit on the thermodynamics. Often, when considering an accelerating black hole, the deficit on one axis (here μ_+) is set to zero, in this case $C = (1-\Delta)/2\Delta$, thus the upper bound is saturated and $\Delta \in [\frac{1}{2}, 1]$. We are interested more generally in how conical deficits impact thermodynamics, so will keep C and Δ arbitrary, within their allowed ranges.

Now turn to the mass formula (2). A check of the thermodynamic expressions in [17] shows that M, S and Q all scale as $1/K$, and J as $1/K^2$. This suggests that scaling each by $1/\Delta$ (or Δ^{-2} in the case of J) is a promising starting point. Some manipulations then reveal the appropriate remaining modifications, and give the mass formula

$$
\begin{aligned}
M^2 = \frac{\Delta S}{4\pi}\Bigg[&\left(1+\frac{\pi Q^2}{\Delta S}+\frac{8PS}{3\Delta}\right)^2\\
&+\left(1+\frac{8PS}{3\Delta}\right)\left(\frac{4\pi^2J^2}{(\Delta S)^2}-\frac{3C^2\Delta}{2PS}\right)\Bigg].
\end{aligned}
\tag{10}
$$

It is then a matter of algebra to confirm that the thermodynamic potentials conjugate to the charges, $T = \frac{\partial M}{\partial S}\big|_{P,J,Q,\mu_\pm}$ etc. correspond to the expressions in [17] and are:

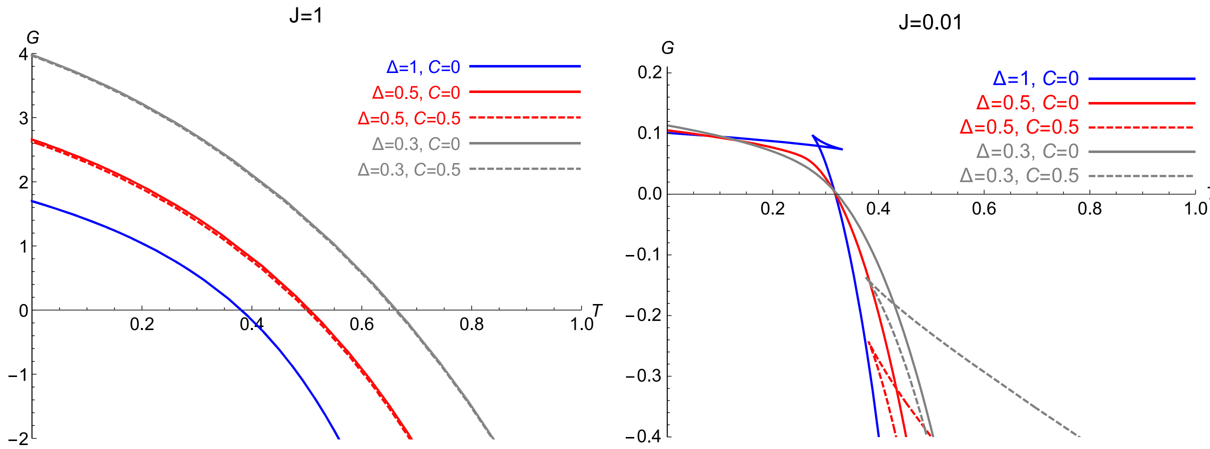

Fig. 1. An illustration of the impact of conical deficit (Δ), and acceleration (C) on the free energy for large and small values of J as labelled, relative to the AdS lengthscale, ℓ, set to unity in these plots. The corresponding plots for Q are qualitatively the same.

$$
\begin{aligned}
V &= \frac{2S^2}{3\pi M}\left[\left(1+\frac{\pi Q^2}{\Delta S}+\frac{8PS}{3\Delta}\right)+\frac{2\pi^2 J^2}{(\Delta S)^2}+\frac{9C^2\Delta^2}{32P^2S^2}\right],\\
T &= \frac{\Delta}{8\pi M}\left[\left(1+\frac{\pi Q^2}{\Delta S}+\frac{8PS}{3\Delta}\right)\left(1-\frac{\pi Q^2}{\Delta S}+\frac{8PS}{\Delta}\right)\right.\\
&\qquad\left.-\frac{4\pi^2 J^2}{(\Delta S)^2}-4C^2\right],\\
\Omega &= \frac{\pi J}{SM\Delta}\left(1+\frac{8PS}{3\Delta}\right),\\
\Phi &= \frac{Q}{2M}\left(1+\frac{\pi Q^2}{S\Delta}+\frac{8PS}{3\Delta}\right),\\
\lambda_\pm &= \frac{S}{4\pi M}\left[\left(\frac{8PS}{3\Delta}+\frac{\pi Q^2}{\Delta S}\right)^2+\frac{4\pi^2 J^2}{(\Delta S)^2}\left(1+\frac{16PS}{3\Delta}\right)\right.\\
&\qquad\left.-(1\mp 2C)^2\pm\frac{3C\Delta}{2PS}\right].
\end{aligned}
\tag{11}
$$

Since everything is now written in terms of the charges, this elucidates the 'chemical' structure of the accelerating black hole, and allows for a more intuitive and natural analysis of the thermodynamics, as well as clarifying some of the new phenomenology of accelerating thermodynamics. We will now illustrate this by making some general observations on the impact of conical deficits, before concluding by presenting a new entropy bound for black holes with conical deficits.

The conical structure of the spacetime appears in two ways: the 'overall' conical deficit, encoded in Δ, that can be present whether or not there is acceleration. $\Delta<1$ means that the spacetime contains a conical deficit, and if $C=0$, the deficit cuts through the whole spacetime, piercing the black hole. Acceleration appears when $C>0$, but unlike angular momentum and charge that contribute to the enthalpy positively, C contributes negatively, indicating an exothermic nature to this particular property. This now opens the possibility of new phenomena in phase space, as, apart from the extremal limit, $T\to 0$, we also potentially have a limit $M\to 0$ that signals a breakdown in the thermodynamical description. This breakdown occurs approximately, though not precisely, at the breakdown of the slow acceleration régime.

Let us begin by exploring the impact of an overall conical deficit, setting $C=0$ and allowing Δ to vary. It might seem that as Δ simply enters as a rescaling parameter, it does not change the qualitative thermodynamics, but the story is more subtle. For example, consider the (canonical ensemble) free energy $G=M-TS$, which in full is

$$
\begin{aligned}
G = \frac{\Delta S}{8\pi M}&\left[\frac{4\pi^2 J^2}{\Delta^2 S^2}\left(3+\frac{16PS}{3\Delta}\right)-4C^2\left(1+\frac{3\Delta}{4PS}\right)\right.\\
&\left.+\left(1+\frac{8PS}{3\Delta}+\frac{\pi Q^2}{\Delta S}\right)\left(1-\frac{8PS}{3\Delta}+\frac{3\pi Q^2}{\Delta S}\right)\right].
\end{aligned}
\tag{12}
$$

For an uncharged, non-rotating, non-accelerating black hole, the magnitude of the free energy is decreased by adding a conical deficit. The Hawking-Page transition [32] therefore still occurs at $T_{HP}=\sqrt{8P/3\pi}=1/\pi\ell$, and the critical point at which the specific heat of the black hole becomes positive (i.e., the minimal temperature that a black hole can have) also remains at $T_m=\sqrt{3}T_{HP}/2$, however, the free energy curves are strongly modified with Δ, and the entropy, or size, of the black hole at each of these critical points is lowered: $S_c=\Delta/8P$, $S_{HP}=3\Delta/8P$.

With the addition of charge and/or rotation, the behaviour is not as simple and depends on the magnitude of the charges Q and J. Recall that for the AdS black hole the free energy surface $G(T,Q)$ (or $G(T,J)$) has a *swallowtail* shape, with a co-existence line between 'small' and 'large' black holes (where size is defined relative to the AdS length scale ℓ) appearing as the charge drops. Introducing a deficit via $\Delta<1$ amplifies the positive contributions of J and Q to G. On the other hand, the contribution from the negative PS/Δ term is also amplified. Put together, the general effect of adding a deficit is to make the behaviour of the free energy that of a 'larger' black hole. Thus, for large charges, the free energy is increased with a deficit, whereas for a smaller charges the effect of a deficit is more nuanced, as seen in Fig. 1.

Now consider adding in acceleration, via the C term. For large black holes with or without charge or angular momentum, the presence of acceleration in itself does not impact strongly on the thermodynamics, rather, it is the fact that acceleration requires an average deficit that modifies the thermodynamics. However, for small black holes things become more interesting. From (12), dropping Q and J lowers the free energy, and for sufficiently low charge can eliminate any regions of positive G, see Fig. 1. The lack of a Hawking-Page transition (for the simple reason that there is no spacetime with 'half' a cosmic string without a black hole) was discussed in [13]. Although this global phase transition does not exist, for charged black holes there exists a local second order phase transition, first noted for non-accelerating black holes in [29]. Generically, for fixed J or Q, there are two values of S at which C_P, the specific heat at constant pressure, diverges, but at some J these two points coalesce, giving criticality. In the non-accelerating case, by expanding analogous quantities to (11) this critical point was seen to possess mean-field exponents [31], the behaviour of a Van-der-Waals gas [6]. Given equations (11), it is straightforward to perform series expansions in much the same manner for the accelerating case, the result yielding the identical Ising-like exponents.

An important new critical phenomenon appears with acceleration, discovered in [30], and that is *swallowtail snapping*, which (10), (11) reveals as occurring precisely because of the exothermic nature of acceleration. Recall that the third law usually provides a lower bound on the entropy, corresponding to the size of the extremal black hole at which $T = 0$. However, in the presence of acceleration, we have a new limit coming from the positivity of M^2. To explore this, abbreviate notation by writing

$$x = \frac{8PS}{3\Delta}\,, \quad \frac{\pi Q^2}{\Delta S} = q\frac{C^2}{x}\,, \quad \frac{\pi J}{\Delta S} = j\frac{C^2}{x} \tag{13}$$

so that

$$M^2 = \frac{\Delta S}{4\pi}\left[\left(1 + x - q_+\frac{C^2}{x}\right)\left(1 + x - q_-\frac{C^2}{x}\right) + 4j^2C^4\frac{(1+x)}{x^2}\right] \tag{14}$$

where $q_\pm = 2 - q \pm 2\sqrt{1-q}$. If $q \le 1$, then the roots $q_\pm$ are real, and there is a range of j for which $M \to 0$ at some x_0. Further, since $T = \frac{1}{2M}\frac{\partial M^2}{\partial S} \propto \frac{1}{M}\frac{\partial M^2}{\partial x}$, this occurs before the extremal limit is reached. For this range of low charge/rotation to acceleration ratios, small black holes are no longer cold, but instead, like their uncharged counterparts, are hot, and have a negative specific heat as the enthalpy tends to zero. As the charge/rotation increases, a critical limit is reached, $q_c(j)$ for which M^2 has a repeated zero, at which T is finite, and above the critical values of charge/rotation, the black hole once again exhibits a swallowtail. This ‘snapping’ of the swallowtail was discovered in [30], although the snapping point could not be determined analytically from the implicit expressions of the thermodynamical variables.

Using the chemical variables, we can readily find the one-parameter family of critical charged, rotating, and accelerating black holes that have zero enthalpy, but finite temperature at the snapping point of the swallowtail. These critical entropies and temperatures are given implicitly by the value of x at which the mass becomes zero, x_0, and C:

$$\begin{aligned}
Q_S^2 &= \frac{3\Delta^2}{8\pi P}(1+x_0)\left[1 + x_0 - \sqrt{(1+2x_0)^2 - 4C^2}\right],\\
J_S^2 &= \frac{1}{2}\left(\frac{3\Delta^2}{8\pi P}\right)^2 (1+2x_0)\Big[2C^2 - (1+x_0)\Big(1 + 2x_0\\
&\qquad - \sqrt{(1+2x_0)^2 - 4C^2}\Big)\Big],\\
T_S &= \sqrt{\frac{2P}{3\pi x_0}}\Big[(4x_0+3)(2x_0+1) - 4C^2\\
&\qquad - 2(1+x_0)\sqrt{(1+2x_0)^2 - 4C^2}\Big]^{1/2},
\end{aligned} \tag{15}$$

where $x_0 \in [\frac{\sqrt{1+4C^2}-1}{2}, \frac{\sqrt{1+12C^2}-1}{3}]$. The lower limit has $J = 0$, $Q^2 = \delta\mu^2\ell^2$, as noted in [30], and the upper limit corresponds to $Q = 0$, $|J| = \Delta^2\ell^2 x_0\sqrt{2x_0+1}/2$, where x_0 takes the upper limit value. What is interesting here is how the overall deficit plays a role in the critical values of Q and J, except for the pure charge snapped swallowtail, where it seems to be the acceleration that is primary driver. As the acceleration decreases, $C \to 0$, and the range and size of x_0 correspondingly decreases, thus the critical temperature for the snapping point also becomes higher, with the critical temperature increasing as x_0 moves towards the lower end of the range (zero angular momentum). The maximum value of acceleration, $C = 1/2$, corresponds to a deficit of 2π along the South axis, and has the lowest values of critical temperature and the largest range of q and j. The absolute lowest critical snapping temperature occurs for $Q = 0$, and is $T = \sqrt{2}/\pi$. An illustration of the snapping swallowtail for a pure charged accelerating black hole is given in Fig. 2.

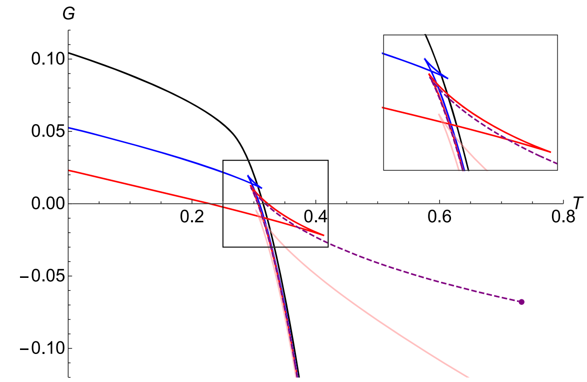

Fig. 2. An illustration of the snapping swallowtail for $J = 0$, $\Delta = 0.5$, $C = 0.25$, and $Q = 0.16, 0.135, 0.127, 0.01$ for the solid black, blue, red, and pink lines respectively, with the critical snapping line $Q_c = C\Delta = 0.125$ in dashed purple. The critical snap point is $(T_c, G_c) = \left(\frac{(2x_0+1)}{2\pi\sqrt{x_0}}, -\frac{\Delta\sqrt{x_0}}{2}(1+2x_0)\right)$, indicated by the solid purple dot.

To conclude our exploration of the accelerating black hole chemistry, consider the *Reverse Isoperimetric Inequality* [4]. The Isoperimetric Inequality is the simple geometric statement that the largest surface area enclosed by a loop of string is when the string is circular, or its higher dimensional equivalent: $(\mathcal{A}/\mathcal{A}_0)^D \ge (\mathcal{V}/\mathcal{V}_0)^{D-1}$, where the subscript 0 indicates the volume or area of a unit sphere. However, for the black hole, the area directly determines the entropy, thus we would expect that for a given volume, a black hole would want to maximise its entropy, which runs counter to the standard inequality that would mean a spherical black hole would be the lowest entropy for that volume, and hence unstable. Analysing a wide range of solutions, Cvetic et al. [4] discovered that black holes always satisfied the *inverse* of this inequality:

$$\left(\frac{\mathcal{A}}{\mathcal{A}_0}\right)^D \le \left(\frac{\mathcal{V}}{\mathcal{V}_0}\right)^{D-1} \tag{16}$$

leading to their *Reverse Isoperimetric Inequality* conjecture.

Let us now explore this inequality in the context of black holes with conical deficits. Note that

$$\begin{aligned}
\frac{4\pi M^2}{\Delta S} &= \left(1 + \frac{\pi Q^2}{\Delta S} + x\right)^2 + 4(1+x)\left(\frac{\pi^2 J^2}{\Delta^2 S^2} - \frac{C^2}{x}\right)\\
&= \left(\frac{3\pi MV}{2S^2} - \frac{2C^2}{x^2}\right)^2 - 4\left(\frac{\pi Q^2}{\Delta S}\right)\left(\frac{\pi J}{\Delta S}\right)^2\\
&\quad - 4\left(\frac{\pi J}{\Delta S}\right)^4 - 4(1+x)\frac{C^2}{x}\,.
\end{aligned} \tag{17}$$

At this point, we spot that the term in brackets on the RHS contains the seed of the isoperimetric ratio:

$$M^2\left(\frac{3V}{4\pi}\right)^2\left(\frac{\pi}{S}\right)^4 \ge \left(\frac{3\pi MV}{4S^2} - \frac{C^2}{x^2}\right)^2 \ge \frac{\pi M^2}{\Delta S} \tag{18}$$

from which we may conclude a *new Reverse Isoperimetric Inequality*, appropriate for spacetimes with a conical deficit:

$$\left(\frac{3V}{4\pi}\right)^2 \geq \frac{1}{\Delta}\left(\frac{\mathcal{A}}{4\pi}\right)^3 \tag{19}$$

with equality iff $C = J = 0$. The larger the conical deficit, the smaller the entropy is with respect to the volume, thus conical defects appear to render black holes sub-entropic.

We have highlighted here some key distinct phenomena displayed by the deficited black hole, and in a companion paper we explore the properties of conical holographic heat engines [33]. Apart from the interesting new phenomenology displayed by accelerating black holes, the ease with which we are able to analyse and uncover this behaviour using the chemical variables is remarkable. Not only have we proved a new Entropy Inequality for black holes with conical deficits, but by writing down a Christodoulou-Ruffini type of mass formula, we have now expressed the mass of a four-dimensional black hole that includes two new charges: Δ and C, or, the average deficit and acceleration. It would be interesting to understand how our expressions relate to those in [34] for an unconfined magnetic flux tube through a black hole. One possibly disturbing aspect of the exothermic impact of acceleration is that it seems to diverge as the pressure tends to zero. This is a feature of the slow-acceleration limit, or, the fact that the time coordinate of a slowly accelerating black hole is proportional to the time coordinate of an asymptotic observer, thus the notion of an isolated mass makes sense. For a black hole whose time coordinate is a boost coordinate, such as the usual vacuum accelerating black hole or Rindler spacetime, alternate thermodynamic parameters, along the lines of the boost mass developed in [35] should be used. Clearly, the thermodynamics and chemistry of these new charges for a black hole merits further study and exploration.

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